

# Disorder-induced light trapping enhanced by pulse collisions in one-dimensional nonlinear photonic crystals

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We use numerical simulations to study interaction of co- and counter-propagating pulses in disordered multilayers with noninstantaneous Kerr nonlinearity. We propose a statistical argument for existence of the disorder-induced trapping which implies the dramatic rise of the probability of realization with low output energy in the structure with a certain level of disorder. This effect is much more pronounced in the case of two interacting pulses than in the single-pulse regime and does not occur in the strictly ordered system at the same intensity of the pulses. Therefore it cannot be explained simply as a result of increase in strength of nonlinear light-matter interaction.

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## I. INTRODUCTION

Since Philip W. Anderson's breakthrough paper [1], study of localization and other matter-wave effects in solid-state disordered systems has become a broad and fruitful field of research. Moreover, the notion of Anderson localization stimulated research of wave phenomena in other contexts, including classical wave dynamics in disordered media and the connections with mesoscopic physics [2–5]. In optics, this interest has led to the experimental observation of the Anderson localization of light in 1990s and 2000s [6–8]. Discussion of subsequent progress in disordered optics and photonics can be found in recent reviews [9, 10].

In this paper, we deal with some aspects of nonlinear optics of disordered photonic structures. For detailed discussion of short-pulse effects (including tail dynamics [11], localization suppression [12, 13], localized solitons formation [14, 15], etc.) in nonlinear disordered systems, see the introduction to my previous paper [16] and references therein. Here we restrict ourselves to referring only to a few recent advances reported in literature. Among them are the observation of the reciprocity breaking effect in nonlinear random medium [17], the parametric amplification of light localization in the random medium with quadratic nonlinearity [18], self-trapping of light in nonlinear waveguide array with coupling disorder [19], nonreciprocal localization in disordered multilayers with magneto-optical materials [20], wave packet spreading in 1D and 2D photonic lattices [21], control of energy transfer in disordered laser resonators [22], etc.

This paper can be viewed as a continuation of the previous work [16] devoted to propagation and self-trapping of ultrashort pulses in disordered one-dimensional photonic crystals with instantaneous and relaxing nonlinearities. Here we consider the collisions of pulses in such structures and search for the possibility of light trapping which cannot be reached in ordered system with the

same parameters. This trapping is fundamentally different from the self-trapping effect in the perfect nonlinear photonic crystals [23] which is destroyed by introduction of disorder. As previously, we consider the regime of strong disorder and strong nonlinearity. We have studied earlier the interaction of co- and counter-propagating pulses in perfect photonic crystals with relaxing nonlinearity [24] and in dense two-level media [25–27]. As far as we know, the influence of disorder on such interaction was not considered in scientific literature yet. The present study makes up for this deficiency.

The paper is structured as follows. In Section II, we give the main equations and briefly discuss the numerical method and the parameters adopted. Sections III and IV are dedicated to the analysis of results obtained for co- and counter-propagating pulses, respectively. The paper is completed with the short Conclusion.

## II. PROBLEM STATEMENT

Let us consider the one-dimensional photonic crystal, i.e. a multilayer structure consisting of two different materials – alternating layers denoted with letters  $a$  and  $b$ . Light is assumed to propagate along the  $z$ -axis which is perpendicular to the layers interfaces. The results reported here are based on numerical solution of the one-dimensional wave equation,

$$\frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 (n^2 E)}{\partial t^2} = 0, \quad (1)$$

where  $E$  is the electric field strength,  $n$  is the medium refractive index which, generally, is a function light intensity  $I = |E|^2$ ,

$$n = n^0(z) + \delta n(I, t, z). \quad (2)$$

Here  $n^0(z)$  is a linear part of refractive index changing periodically along the structure. Since we deal with non-instantaneous nonlinearity, the nonlinear contribution  $\delta n$  must take into account the relaxation process which, for

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definiteness, will be described by the Debye model [28],

$$t_{nl} \frac{d\delta n}{dt} + \delta n = n_2 I, \quad (3)$$

where  $n_2$  is the cubic (Kerr) nonlinear coefficient, and  $t_{nl}$  is the relaxation time. For the disordered periodic structure, we assume the random variations of thicknesses of layers  $a$  and  $b$  as follows,

$$d_{a,b} = d_{a,b}^0 + \Delta d(\xi - 1/2), \quad (4)$$

where  $d_{a,b}^0$  are the mean values of thicknesses,  $\Delta d$  is the amplitude of disorder, and  $\xi$  is the random quantity uniformly distributed in the range  $[0, 1]$ .

We solve numerically Eqs. 1-4 using the method developed in the previous publications [16, 23]. As previously, we do not mean any specific materials, since our aim is to study the qualitative and general aspects of light interactions with periodic disordered structures. Therefore, for our calculations, we adopt the parameters of the model from Ref. [16]:  $d_a^0 = 0.4$  and  $d_b^0 = 0.24 \mu\text{m}$ ,  $n_a^0 = 2$  and  $n_b^0 = 1.5$ . The envelope of the pulse at the input of the photonic structure is supposed to have the Gaussian shape,  $A(t) = A_0 \exp(-t^2/2t_p^2)$ , where  $t_p$  is the pulse duration, and  $A_0$  is the amplitude of the electric field. Further we assume  $t_p = 50$  fs and the central wavelength  $\lambda_c = 1.064 \mu\text{m}$ , so that the carrier frequency lies just outside the band gap of the perfect multilayer [16]. Finally, we restrict ourselves to the structure with nonlinear  $b$  layers only. This is justified, because light concentrates in these layers when, as in our case, we deal with the high-frequency edge of the band gap [29]. The strength of nonlinearity ( $n_2 I_0 = n_2 |A_0|^2 \sim 0.01$ ) is taken to be large enough to strongly influence the pulse characteristics. This allows to consider comparatively short systems, namely  $N = 50$  periods in our calculations. Construction of such photonic crystals seems to be quite feasible for modern technology. Though we do not mean any specific materials, linear layers may be formed by glass, while for nonlinear layers one can use polymer materials possessing high nonlinearity and fast relaxation [30]. However, as far as we know, such photonic crystals possessing relaxing nonlinearity and disorder simultaneously were not realized experimentally yet. Therefore, our study can be considered as a proposal for building such new optical systems as well.

Thus, we consider the interplay of strong disorder and strong nonlinearity. Generally, this interplay can be studied on the short timescale (pulse shape transformation) and at long times (pulse tail transformation as an evidence for the Anderson localization) as was done in the previous work [16]. In this paper, we deal with the collisions of pulses in the disordered photonic crystals. Since the behavior of the tail and the Anderson localization seem to be insensitive to the number of pulses, we will focus on the shape transformations of the colliding pulses and, in particular, on the possibility to induce light trapping by using the collisions of co- and counter-propagating pulses.

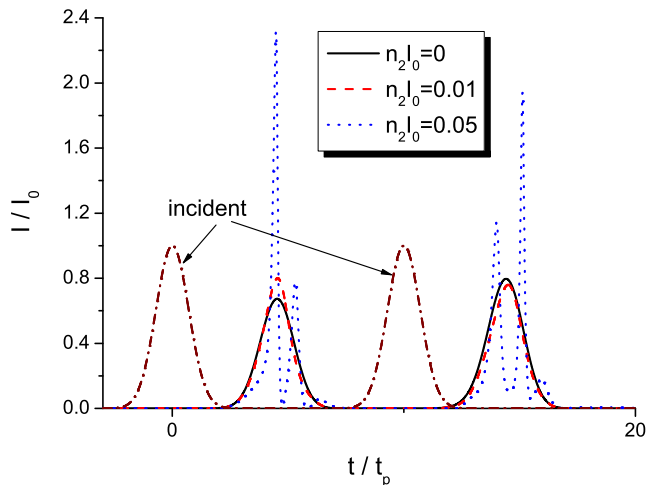


FIG. 1. (Color online) The profiles of co-propagating pulses transmitted through the perfect (ordered) photonic crystal with and without nonlinearity.

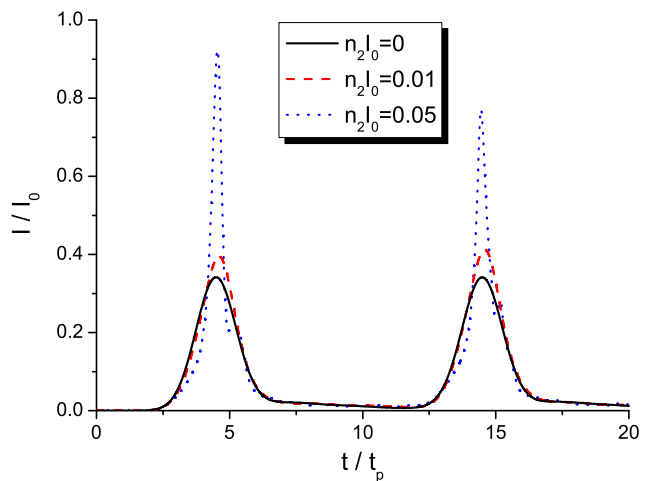


FIG. 2. (Color online) The same as in Fig. 1 but for the disordered structure with  $\Delta d = 0.05 \mu\text{m}$ . The curves are averaged over 50 realizations.

### III. CO-PROPAGATING PULSES

First, let us consider the situation of two co-propagating pulses launched into the structure with some interval one after another. This interval must be not too large for the pulses to interact effectively with each other and not too small so that we can talk about separate pulses. In our calculations, we assume the interval of  $10t_p$  between the peaks of the incident pulses. We start with the profiles of the pulses transmitted through the perfect (ordered) photonic crystal (Fig. 1). It is seen that the pulses have different peaks even in the linear case. This means that the interpulse interval is short enough to provide effective energy interchange between them. Perhaps, in the linear case, some residual radiation of the first pulse joins the second one, so that its

TABLE I. The data on realizations in the case of two pulses co-propagating through the disordered photonic crystal. The parameters  $n_2 I_0 = 0.05$  and  $t_{nl} = 10$  fs are used, if the other is not stated. The simulation time is  $100t_p$ . (The notations are used as follows:  $\bar{T}$  is the average transmission,  $\bar{R}$  the average reflection,  $\bar{W}$  the average output energy, i.e.  $\bar{T} + \bar{R}$ ;  $N_{90}$  the number of realizations with the output energy  $W > 0.9$ ,  $N_{80}$  the number of realizations with  $0.8 < W < 0.9$ ,  $N_{70}$  the number of realizations with  $0.7 < W < 0.8$ ,  $N_{60}$  the number of realizations with  $0.6 < W < 0.7$ ,  $N_{50}$  the number of realizations with  $W < 0.6$ ;  $N_t$  the total number of realizations;  $W_{min}$  the minimal output energy  $W$  among the realizations.)

$\Delta d$	$T$	$R$	$W$	$N_{90}$	$N_{80}$	$N_{70}$	$N_{60}$	$N_{50}$	$N_t$	$W_{min}$
0	0.618	0.359	0.976	1	-	-	-	-	1	0.976
0.01	0.588	0.375	0.963	94	6	-	-	-	100	0.809
0.02	0.548	0.379	0.927	77	14	6	2	1	100	0.558
0.03	0.512	0.413	0.925	79	7	8	5	1	100	0.527
0.04	0.479	0.441	0.920	74	14	8	2	2	100	0.481
0.05	0.419	0.509	0.928	82	4	7	5	2	100	0.516
0.06	0.361	0.587	0.948	82	13	-	5	-	100	0.625
0.07	0.309	0.644	0.953	83	13	3	1	-	100	0.653
0.08	0.272	0.690	0.962	89	7	1	3	-	100	0.644
0.09	0.222	0.748	0.970	89	11	-	-	-	100	0.824
0.10	0.192	0.787	0.979	96	2	2	-	-	100	0.793
0.05 (single pulse)	0.450	0.521	0.971	93	5	1	1	-	100	0.649
0.05 ( $n_2 I_0 = 0.01$ )	0.45	0.5489	0.9989	50	-	-	-	-	50	0.9975
0.05 ( $t_{nl} = 0$ )	0.5136	0.4782	0.9918	50	-	-	-	-	50	0.956

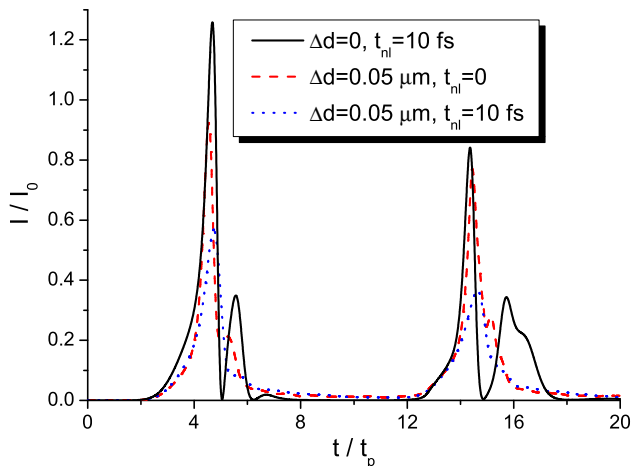


FIG. 3. (Color online) The profiles of co-propagating pulses transmitted through the ordered and disordered photonic crystals with and without nonlinearity relaxation. The non-linearity coefficient is  $n_2 I_0 = 0.05$ . The curves are averaged over 50 realizations.

intensity grows. This simple picture is not applicable for the more complicated nonlinear case. In nonlinear structure, the first pulse is stronger compressed (more intense) than the second one. Figure 2 shows the changes in the profiles due to disorder with  $\Delta d = 0.05 \mu\text{m}$ . In the linear case, the averaged transmitted pulses seem to be almost identical, i.e. on average, the distribution of energy between the pulses is uniform. This uniformity is broken as a result of nonlinearity introduction: the first pulse tends to be more powerful than the second one. Now we can add the relaxation of nonlinearity and study its influence on the averaged profiles of the co-propagating pulses (Fig. 3). It is seen that addition of relaxation to

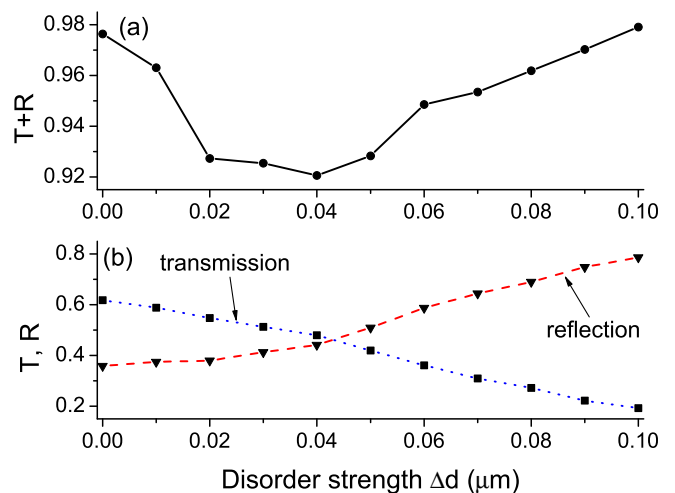


FIG. 4. (Color online) The dependencies of (a) the average total output and (b) the average transmission and reflection on the strength of disorder. The nonlinearity coefficient is  $n_2 I_0 = 0.05$ , the relaxation time is  $t_{nl} = 10$  fs. The averaging was made over 100 realizations.

the disordered structure results in further decrease of the intensity of transmitted pulses.

What is the reason for this decrease? Does it mean simply strengthening of reflection? The detailed study shows that the answer is “no”. According to the data shown in Table I, the average transmission  $\bar{T}$  (the part of total light energy transmitted through the structure in the time  $100t_p$  and averaged over realizations) drops due to the relaxation from 0.514 to 0.419 (remind that we consider the disorder strength  $\Delta d = 0.05 \mu\text{m}$ ). At the same time, the reflection  $\bar{R}$  averaged over realizations grows from 0.478 only to 0.509. This means that

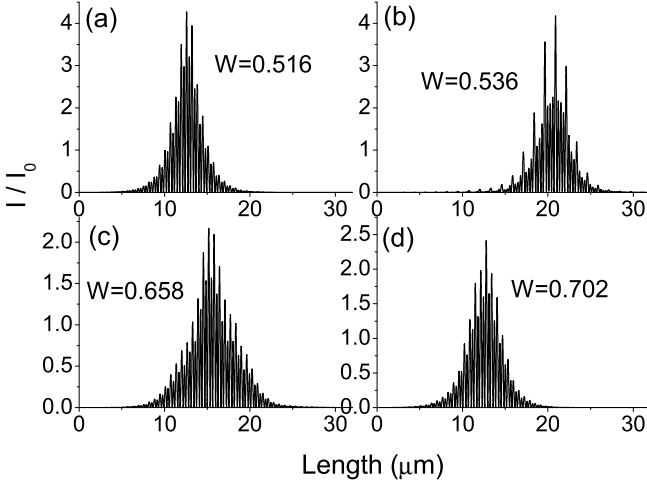


FIG. 5. The intensity distributions (at  $t = 99t_p$ ) for several realizations of pulses co-propagating in the structure with the nonlinearity coefficient  $n_2 I_0 = 0.05$ , relaxation time  $t_{nl} = 10$  fs, and the disorder strength  $\Delta d = 0.05 \mu\text{m}$ .

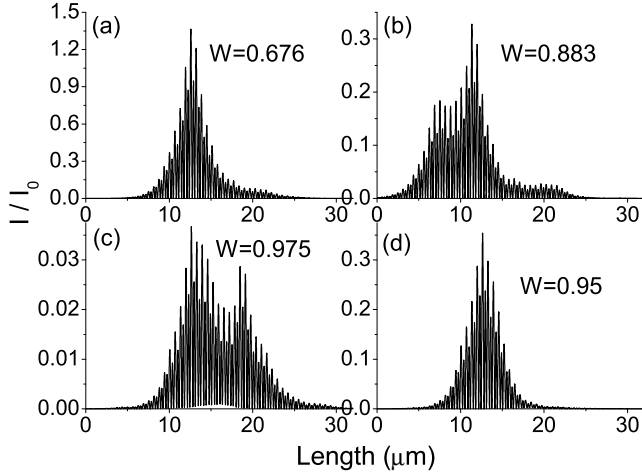


FIG. 6. (a)-(c) The intensity distributions for several realizations of single pulse propagating in the disordered structure with the same parameters as in Fig. 5. Panels (a)-(b) correspond to the same realizations as in Fig. 5(a) and Fig. 5(b), panel (d) shows the two-pulse distribution for the realization of panel (c).

the total average output  $\bar{W}$  (sum of transmission and reflection) decreases from almost unity to 0.928, i.e. *on average* more than 7% of the input energy remains inside the structure due to the relaxation of nonlinearity. We further explored how the average output energy depends on the disorder strength. The resulting curves presented in Fig. 4 show that, as it would be expected of the disordered media, the transmission decreases and reflection increases with the growing  $\Delta d$ . However, these two processes do not compensate each other, so that the dip in the curve for the total output energy appears. The minimum of  $\bar{W}$  occurs at  $\Delta d = 0.04 \mu\text{m}$  and amounts to

about 0.92.

The data on average output implies that there may act the mechanism analogous to the self-trapping effect reported in the previous publications [16, 23]. This assumption is justified by consideration of concrete realizations; the intensity distributions (at the time instant  $t = 99t_p$ ) along the structure for some of realizations with comparatively low output  $W$  are shown in Fig. 5. These distributions correspond to the residual radiation left in the disordered multilayer after passage of both pulses. It is seen that the width and peak intensity of the distributions strongly depends on the characteristics of the concrete realization. For example, comparison of Fig. 5(a) and 5(b) shows that, even at approximately the same value of the output  $W$ , the distributions can strongly differ from each other. As a rule, however, decrease in  $W$  is accompanied by raise of the peak intensity and by narrowing of the distribution. One can compare these two distributions with the residual intensity distributions after passage of a single pulse through the structure with the same parameters (the same realization) shown in Fig. 6(a) and 6(b). It is seen that the output is much greater in the case of the single pulse, i.e. a significant part of energy is trapped inside the system as a result of the interaction of the pulses. The difference between the results in Fig. 6(a) and 6(b) can be compared with the difference of distributions in Fig. 5(a) and 5(b): in the panel (b), light is trapped near the exit of the structure, so that the light storage is not so stable as in the panel (a). Finally, in Fig. 6(c) and 6(d), we plot the distributions of single and double pulse residual intensities for the “usual” (large-output) realization. Though the difference in the absolute value of  $W$  is small, the distribution in the case of co-propagating pulses has a characteristic symmetric shape implying the formation of the (quasi)stable “trap”.

Obviously, the examples discussed above can be treated as an evidence of light trapping enhanced by interaction of two co-propagating pulses. The statistical confirmation of this effect is given in Table I which shows the number of realizations with the outputs  $W$  in the certain ranges for different disorder strengths  $\Delta d$ . It is seen that, in accordance with the data of Fig. 4, the number of low-output realizations grows with increasing  $\Delta d$ , reaches maximum at  $\Delta d = 0.04 \mu\text{m}$  and then starts to decrease. At  $\Delta d = 0.1 \mu\text{m}$ , we have  $W > 0.9$  for almost all realizations. The same fall and rise is characteristic for the value of minimal output among the realizations at a given disorder strength (see the last column of the table). The last three strings of Table I allow us to compare the case of co-propagating pulses with the cases of a single pulse, comparatively weak nonlinearity and no relaxation at the same disorder (namely,  $\Delta d = 0.05 \mu\text{m}$ ), respectively. This comparison shows that the two-pulse scheme allows to strongly increase the efficiency of light trapping inside the disordered photonic crystal.

Further, we have studied the propagation of a single pulse containing the same energy as two interacting

pulses considered above, i.e. we dealt with the pulse of the peak intensity  $2I_0$ . Can the results on trapping enhanced by two interacting pulses be compared with this single pulse case? Our calculations show that the probability of high-intensity pulse trapping in disordered structure with  $\Delta d = 0.05 \mu\text{m}$  is much larger than in two-pulse scheme of Table I: we have the average output  $\bar{W} \approx 0.774$ ,  $W_{\min} = 0.416$  and the number of realizations with  $W < 0.6$  as large as 21 (from the total number of 100). However, such great efficiency of trapping has simple explanation: high-intensity pulse trapping can be observed already in the ordered system giving  $W = 0.640$ . This is the fundamental difference with the situation reported above for the pulses of lower intensities which can be trapped only in the presence of disorder. For the high-intensity pulse, the situation seems to be inverted: the disorder leads to some degradation of trapping, since the average output  $\bar{W}$  is higher than the output for the ordered photonic crystal. Thus, if we want to have the effect of disorder on interacting pulses discussed in this section, the intensity of pulses should be not too high: under this condition, the self-trapping can be observed neither for the single pulse in the disordered structure nor for the co-propagating pulses in the ordered structure. Further, we will deal only with the pulses of appropriate intensity.

The results of calculations reported in this section make it clear that there is an optimal level of disorder for observation of trapping of energy of co-propagating pulses. This fact along with the absence of trapping in the perfect (ordered) structure is the reason for us to call this effect *the disorder-induced light trapping* in the photonic crystal.

#### IV. COUNTER-PROPAGATING PULSES

In this section, we consider another situation when the interacting pulses are counter-propagating. We restrict ourselves to the symmetric scheme when both pulses are identical. This means that, on average, reflection and transmission through the photonic crystal should be equal, i.e.  $\bar{T} = \bar{R} \approx 0.5$ . Here the convention is used as follows: We call "reflection" the part of total energy outgoing from, say, the left edge of the structure, i.e. it is generated by the reflected light of the pulse propagating from left to right (LR) and transmitted light of the pulse propagating from right to left (RL); hence, for "transmission" we have the energy part from the right edge, i.e. transmission of LR pulse plus reflection of RL pulse.

Is it possible to observe the disorder-induced trapping as a result of collision of counter-propagating pulses? The answer is not obvious, since the time of interaction in this scheme seems to be much shorter than for co-propagating pulses moving side by side along the whole length of the structure. We performed calculations for different disorder strengths in the manner of previous section. The statistics of realizations is represented in Table II, while

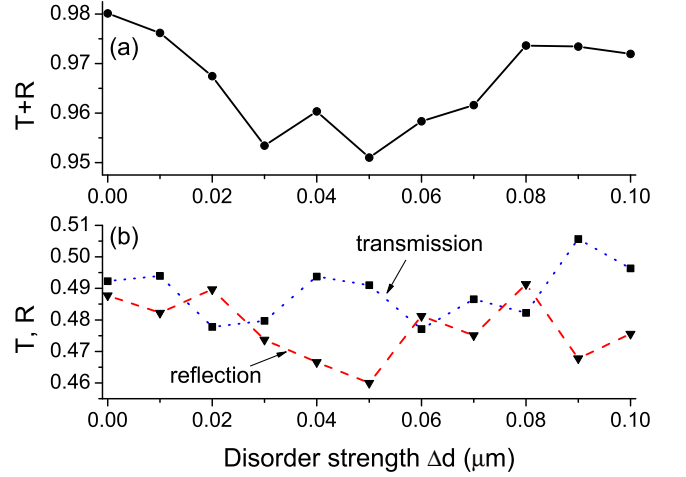


FIG. 7. (Color online) The dependencies of (a) the average total output and (b) the average transmission and reflection on the strength of disorder in the case of counter-propagating pulses. The nonlinearity coefficient is  $n_2 I_0 = 0.05$ , relaxation time  $t_{nl} = 10$  fs. The averaging was made over 100 realizations.

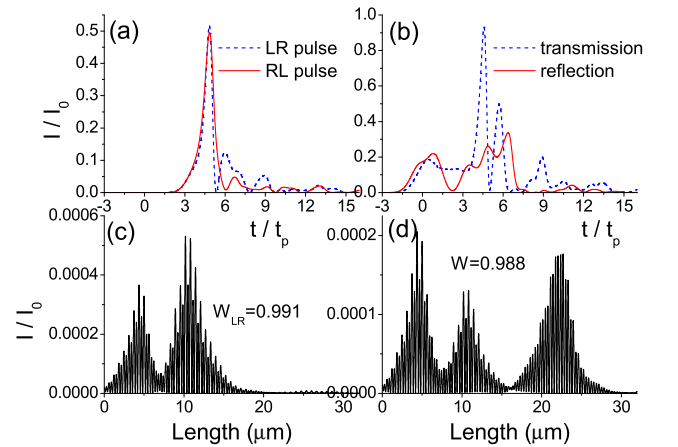


FIG. 8. The example of specific realization with  $W = 0.988$  in counter-propagating regime. (a) Profiles of single LR and RL pulses transmitted through the structure with a given disorder, (b) profiles of transmitted and reflected light when both pulses are launched into the structure, (c) and (d) intensity distributions (at the time instant  $t = 99t_p$ ) corresponding to the situations of (a) and (b), respectively. The nonlinearity coefficient is  $n_2 I_0 = 0.05$ , relaxation time  $t_{nl} = 10$  fs, and the disorder strength  $\Delta d = 0.05 \mu\text{m}$ .

Fig. 7 shows the average values of output energy. It is seen that transmission and reflection vary around the same average level, but the total output  $\bar{W}$  and the minimal output energy  $W_{\min}$  have the lowest values in the range  $\Delta d = 0.03 - 0.05 \mu\text{m}$ . This is in accordance with the results of the previous section [see Fig. 4(a)], though the maximal part of energy remaining inside the photonic crystal drops from about 7% (for co-propagating pulses) to less than 5% (for counter-propagating pulses). The



TABLE II. The data on realizations in the case of two pulses counter-propagating through the disordered photonic crystal. The parameters  $n_2 I_0 = 0.05$  and  $t_{nl} = 10$  fs are used, if the other is not stated. The simulation time is  $100t_p$ .

$\Delta d$	$T$	$R$	$W$	$N_{90}$	$N_{80}$	$N_{70}$	$N_{60}$	$N_{50}$	$N_t$	$W_{min}$
0	0.492	0.488	0.980	1	-	-	-	-	1	0.980
0.01	0.494	0.482	0.976	100	-	-	-	-	100	0.952
0.02	0.478	0.490	0.968	98	2	-	-	-	100	0.826
0.03	0.480	0.474	0.954	92	4	1	2	1	100	0.578
0.04	0.494	0.467	0.960	90	8	2	-	-	100	0.744
0.05	0.491	0.460	0.951	87	7	4	1	1	100	0.594
0.06	0.477	0.481	0.958	90	7	2	1	-	100	0.674
0.07	0.487	0.475	0.962	89	8	3	-	-	100	0.748
0.08	0.482	0.491	0.973	94	6	-	-	-	100	0.824
0.09	0.505	0.468	0.973	98	1	1	-	-	100	0.743
0.10	0.496	0.476	0.972	95	2	1	2	-	100	0.641
0.05 (single pulse)	0.450	0.521	0.971	93	5	1	1	-	100	0.649
0.05 ( $n_2 I_0 = 0.01$ )	0.484	0.515	0.999	50	-	-	-	-	50	0.997
0.05 ( $t_{nl} = 0$ )	0.500	0.494	0.994	50	-	-	-	-	50	0.957

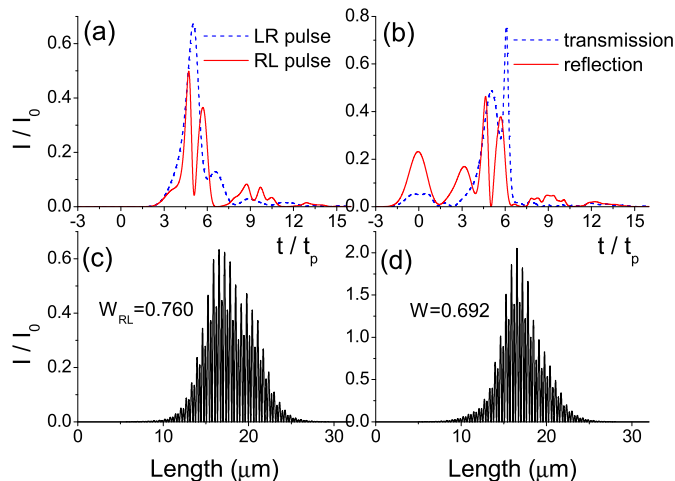


FIG. 9. The same as in Fig. 8 but for another realization with  $W = 0.692$  in counter-propagating regime.

value of the disorder strength at which the output minimum occurs ( $\Delta d = 0.05 \mu\text{m}$ ) is, perhaps, connected with the parameters of the structures considered, in particular with the average thicknesses of the layers: the amplitude of disorder should be high enough comparing with these thicknesses to observe influence of disorder, but not too high, so that the structure can still be considered as periodic on average.

The appearance of disorder-induced trapping is confirmed by the statistics of realizations shown in Table II: the number of realizations with low output first increases with disorder strength and then decreases. The same is true for the minimal output  $W_{min}$  among the realizations. The last three strings of the table calculated for control corroborate the importance of collisions, high enough nonlinearity and presence of relaxation to obtain effective trapping. The effect is less pronounced in comparison with the case of co-propagating pulses, perhaps, because of lower interaction time as mentioned above.

Let us consider two typical realizations. One of them shown in Fig. 8 does not reveal any substantial trapping due to the collision (the output energy is  $W = 0.988$ ). Another realization demonstrated in Fig. 9 is characterized by trapping of approximately 30% of energy of the colliding pulses. If there is only one pulse (either LR or RL), then transmission is almost identical for both directions of propagation in the first case [see Fig. 8(a)]. The collision generally breaks this symmetry, so that transmission and reflection (in the sense discussed above) have very different intensity profiles [Fig. 8(b)]. On the contrary, in the strong trapping regime, there is no transmission symmetry even in the absence of counter-propagating pulse [Fig. 9(a)]. Finally, we should consider the distributions of residual light intensity along the structure [panels (c) and (d)]. In the realization shown in Fig. 8, collision does not qualitatively change the distribution: there is still several peaks of very low intensity. Fundamentally another situation is seen in Fig. 9: the collision results in formation of a single high-intensity bell-shaped peak which can with every reason be called “the trap”. Figure 9(c) shows the distribution only for the RL pulse with  $W_{RL} = 0.76$ , while for LR pulse we have  $W_{RL} = 0.993$ , so that in this last instance there is no any substantial light trapping. The trap becomes more intensive and symmetric in shape (and, hence, more stable) in the case of colliding pulses [Fig. 9(d)]. Of course, there is possibility that the collision will make trapping less effective, but, according to the statistics discussed above, the inverse situation is more probable.

## V. CONCLUSION

In this paper, we predict the possibility of observing light trapping enhanced by collisions of pulses in disordered photonic crystals with relaxing cubic nonlinearity. Since there is an optimal value of disorder, we call this effect the disorder-induced trapping. At very

low (or no) disorder strengths and at very high disorder strengths, the probability of effective light trapping (measured as a number of realizations with low output energy) is strongly suppressed. It is also necessary to have high enough nonlinearity coefficients with nonzero relaxation times, but not too high so that the purely nonlinear trapping to be absent in the ordered case. Though, at these conditions, some part of energy can be trapped even in the single-pulse regime, interaction of pulses (either co-propagating or counter-propagating) strongly increase the number of realizations with effective trapping. Our observations can be considered as the preliminary

report on the possibility of this effect. More investigations are needed to search for the optimal parameters or to study the effect with larger number of interacting pulses.

## ACKNOWLEDGMENTS

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